October Math Gems

PROBLEM OF THE WEEK 11

§1 Problems

Problem 1.1. Let n be a positive integer s.t

$$\frac{1}{9\sqrt{11}+11\sqrt{9}}+\frac{1}{11\sqrt{13}+13\sqrt{11}}+\cdots+\frac{1}{n\sqrt{n+2}+(n+2)\sqrt{n}}=\frac{1}{9}$$

find the value of n.

Problem 1.2. What is the remainder when 9^{1190} is divided by 11?

Problem 1.3. prove that $7|(2222^{5555}+5555^{2222})$.

Problem 1.4. When a positive integer n is divided by $5, 7, 9, 11$, the remainders are $1, 2, 3, 4$ respectively. Find the minimum value of n.

Problem 1.5. Prove that $7^n - 1$is always divisible by 6.

Problem 1.6. Let N be a positive for which the sum of its smallest factors is 4 and the sum of the largest factors is 204. Find the value of $\frac{N}{3}$.

Problem 1.7. Let a, b are relatively prime integers with $a > b > 0$ and

$$\frac{a^3-b^3}{(a-b)^3}=\frac{73}{3}$$

What is the value of $a - b$?

Problem 1.8. A positive integer n is nice if a positive integer m was exactly four positive divisors (including 1 and m) s.t the sum of four divisors is equal to n. How many numbers in the set $\{2010, 2011, 2012, \ldots, 2019\}$ are nice?

Problem 1.9. Find the sum of digits of the largest possible integer n such that $n!$ ends with exactly 100 zeros.

Problem 1.10. The number of numbers of the form $30a0b03$ that are divisible by 13 where a, b are digits is?

Problem 1.11. The digits $1, 2, 3, 4, 5, 6, 7$, and 9 are used to form four two-digit prime numbers, with each digit used exactly once. Then the sum of these four primes is given by $10A$. Find the value of A.

Problem 1.12. Find the number of positive divisors of

$$(2008^3 + 3 \times 2008 \times 2009 + 1)^2$$

Problem 1.13. Prove that the equation $a^2 + b^2 - 8c = 6$ has no integer solution.

Problem 1.14. There are N numbers of positive integers not exceeding 2001 and are multiples of 3 or 4 but not 5. Then find the value of

$$\frac{N-1}{50}$$

Problem 1.15. Let n be a 5-digit number, and let r and q be a quotient and remainder, respectively, when n is divided by 100. Let the total number of value of n for which $q + r$ divisible by 11 is P. Then find the last two digits of P.

Problem 1.16. Let x and y be a two-digit integers such that y is obtained by reversing the digits of x. Suppose that the integers x and y satisfy $x^2 - y^2 = m^2$ for positive integer m. What is the value of

$$\frac{x+y+m}{11}$$

?

Problem 1.17. How many positive integer n does $1 + 2 + 3 + 4 + \cdots + n$ evenly divide from $6n$?

Problem 1.18. If N is the number of four-digit positive integers with at least one digit that is a 2 or a 3 then find the sum of the digits in N.

Problem 1.19. What is the sum of the digits of the square 111 111 111 ?

Problem 1.20. If a, b, c are positive real numbers such that

$$ab + a + b = bc + b + c = ca + a + c = 35$$

Then the value of $(a + 1)(b + 1)(c + 1) = x^y$. Find the product of x and y.
